6.2a Integration by Substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

hard easy

$\int f(x)dx = \int g(u)du$

$\int \sin(x)e^{\cos(x)}dx = \int \cancel{\sin x}\, e^u \frac{du}{-\cancel{\sin x}}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\frac{du}{-\sin x} = dx$

$-\int e^u du = -e^u + c$

check: take der

$-e^u$

$\boxed{-e^{\cos x} + c}$

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$\int x^2\sqrt{5+2x^3}\,dx = \int \cancel{x^2}\sqrt{u}\,\frac{du}{6\cancel{x^2}} = \frac{1}{6}\int\sqrt{u}\,du$

$u = 5+2x^3$

$\frac{du}{dx} = 6x^2$

$du = 6x^2 dx$

$\frac{du}{6x^2} = dx$

$= \frac{1}{6}\int u^{1/2}du = \frac{1}{6}u^{3/2}\cdot\frac{2}{3} + c$

$\boxed{= \frac{1}{9}(5+2x^3)^{3/2} + c}$

$\frac{u^{3/2}}{3/2}$ $\frac{u^{n+1}}{n+1}$

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$\int \cot(7x)dx = \int \frac{\cos(7x)}{\sin(7x)}dx$

$u = \sin(7x)$

$\frac{du}{dx} = 7\cos 7x$

$du = 7\cos 7x\, dx$

$\frac{du}{7\cos 7x} = dx$

$\int \frac{\cancel{\cos 7x}}{u}\frac{du}{7\cancel{\cos 7x}}$

$\frac{1}{7}\int\frac{1}{u}du$

$\frac{1}{7}\ln|u| + c$

$\frac{1}{7}\ln|\sin 7x| + c$

$\boxed{\int\frac{1}{u}du = \ln|u| + c}$

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$\int \frac{dx}{\cos^2 2x}$

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$\int \cot^2 3x\, dx$

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$\int \cos^3 x\, dx$

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Definite Integrals

$\int x\,dx = \frac{x^2}{2} + c$

$$\boxed{\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx} = \boxed{\int_0^{\sqrt{3}} u\,du} = \frac{u^2}{2}\Big|_0^{\sqrt{3}}$$

$u = \tan x$

$x = 0 \quad u = \tan 0 = 0$

$x = \frac{\pi}{3} \quad u = \tan\frac{\pi}{3} = \sqrt{3}$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\, dx$

$\frac{\sqrt{3}^2}{2} - \frac{0^2}{2} = \boxed{\frac{3}{2}}$

$\frac{u^2}{2} = \frac{(\tan x)^2}{2}$

$\frac{(\tan x)^2}{2}\Big|_0^{\pi/3} = \frac{\left(\tan\frac{\pi}{3}\right)^2}{2} - \frac{(\tan 0)^2}{2}$

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$\int_0^1 \frac{x}{x^2 - 4}\,dx$

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